Concours de l'Association mathématique du Québec
Niveau collégial

Wednesday, February 8 2023

TO THE COMPETITORS :

This is a competition rather than an examination; it is therefore natural that you may find certain questions difficult and that you may not be able to answer them all. Marking shall be done confidentially and will take into account various elements : approach, precision, clarity, rigour, originality and, in the case of a partial answer, a sketch of a viable solution.

We thank you and congratulate you for your interest in mathematics. Good luck!

Note *:The use of any kind of calculator is prohibited.*

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1. **Median crossing** Show that the tangents lines of $y = ax^2 + bx + c$ at any points p and q on the $x-$axis intersect at a point whose x coordinate is $\dfrac{p+q}{2}$ (halfway between p and q).

2. **Parallelogram** Anik has two blue sticks of equal length as well as two red sticks, also of equal length. She makes a parallelogram with an angle of $75°$. She remarks that the sum of the length squared of its diagonals is 98. She then forms a rectangle with her four sticks. What is the lenght of a diagonal of this rectangle ?

3. **The hare and the turtle** The hare and the tortoise are having a race. They both start at the same time.

   The tortoise has a constant velocity of 1.5 km/h while the hare's velocity (in km/h) as a function of time (measured in hours) is given by the degree 2 polynomial t^2+bt+c where b and c are constants to be determined. The hare's initial velocity and final velocity are 5 km/h and 10 km/h, respectively. Against all odds, both participants cross the finish line at the exact same time.

   How many kilometers did the tortoise run during the race ?

4. **This year's composition**

   Consider the polynomial function $f(x) = 2x^2 - 1$. Compute and simplify as much as possible the expression

   $$\underbrace{f \circ f \circ \cdots \circ f}_{2023}\left(\cos\left(\frac{\pi}{2^{2023}}\right)\right).$$

   Reminder : $f \circ g(x)$ means $f(g(x))$, $f \circ g \circ h(x)$ means $f(g(h(x)))$, etc...

5. **Mister Impatient**

   Every morning, mister Impatient goes to his bus stop at a random time between $6:00$ am and $7:00$ am. He waits for 12 minutes and if the bus has not arrived yet, he starts walking to work. The bus also arrives at a random time between $6:00$ am and $7:00$ am. In any given morning, what is the probability that mister Impatient gets on the bus on its way to work ?

   Note : The arrival times of both mister Impatient and the bus are uniformly distributed across $6:00$ *am to* $7:00$ *am*

6. **Nothing in common** Give the number of positive integers smaller than $2023 = 7 \cdot 17^2$ which have no prime common factors with 2023.