Concours de l'Association mathématique du Québec
Niveau collégial

Le mercredi 12 février 2025

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1. **The largest known prime**

   The largest prime number currently known to humanity was discovered on October 12, 2024. The number is

   $$2^{136\,279\,841} - 1.$$

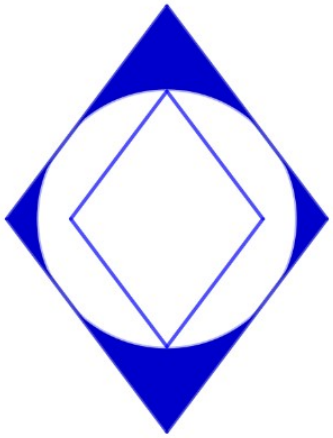
   What digit appears in the units position for this number ?

2. **Area = perimeter**

   Consider two triangles, one with sides measuring 5, 12 and 13 units, and the other with sides measuring 6, 8 and 10 units. Show that these are the only two right-angled triangles of integer lengths for which the perimeter, when measured in units, has the same numerical value as its area when measured in units squared.

3. **The little rhombus**

   Consider a circle inscribed within a rhombus whose diagonals are of length 15 and 20. Inside this circle, one can inscribe a smaller rhombus that is similar to the first one. Calculate the area of the smaller rhombus.

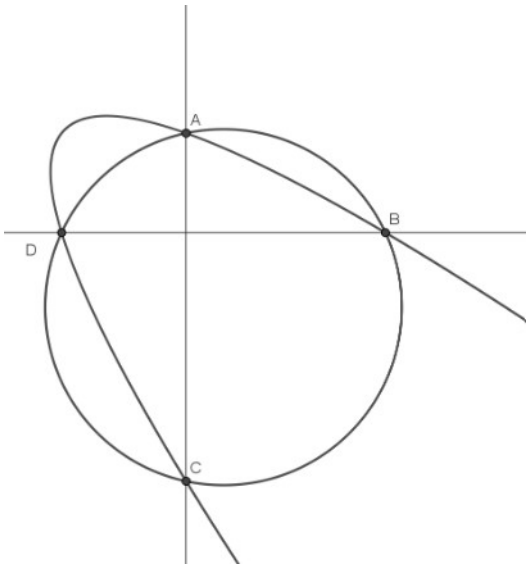


4. **The parabola with a circular heart**

Consider the oblique parabola given by the equation

$$2x^2 + 4xy + 2y^2 - 3x + 6y = 20.$$

The parabola intersects the coordinate axes at four points, shown as A, B, C and D in the diagram. Find the equation of the circle passing through these four points.



5. **Inverted derivatives**

The function $f(x) = x + 2x^2 + 3x^3$ has an inverse, which is denoted by f^{-1}. Calculate the following :

(a) $f^{-1}(0)$;

(b) $\dfrac{d\,(f^{-1})}{dx}(0)$;

(c) $\dfrac{d\,(f^{-1})}{dx}(6)$;

(d) $\dfrac{d^2\,(f^{-1})}{dx^2}(6)$.

6. **The Mathville Pixels**

Nine players of a baseball team are seated at a round table. They are brought 9 caps, of which 3 are red, 3 are green and 3 are blue. The caps must be distributed in such a way that each player gets one with a different colour than those of their two neighbours. In how many different ways can one distribute the caps to these 9 players ?