# Concours de l'Association mathématique du Québec
# Niveau collégial

**Le mercredi 14 février 2024**

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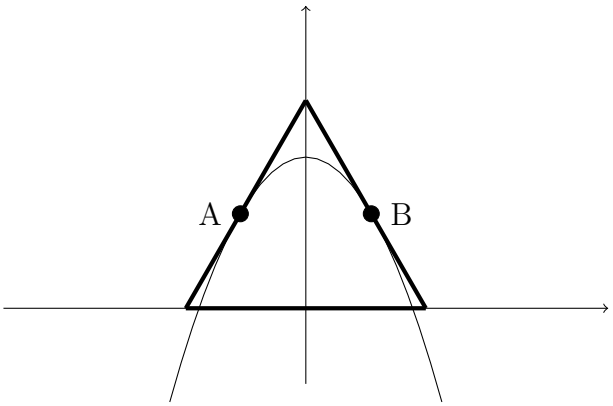
1. **Fair and square !**

   We would like to construct a large square room whose floor is completely covered by whole tiles that are rectangles of dimension $46 \times 88$. Find the minimal (non-zero) dimension of the room.

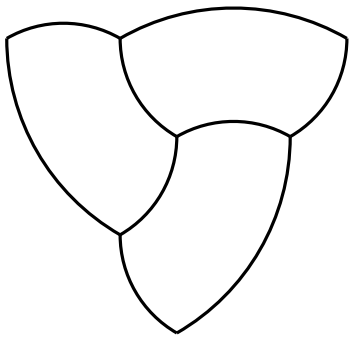
2. **Equilateral tangency**

   Consider the function $f(x) = 1 - x^2$. Determine the two points $A$ and $B$ such that the tangents to the graph form an equilateral triangle with a part of the $x$-axis as base.




3. **The shield**

   A shield is composed of three identical pieces that slot into each other perfectly. The curves on the shield are circular arcs. The small circular arcs are all identical and arise from circles of radius 1. Similarly, the big circular arcs are identical and arise from circles of radius 2. Each of the three pieces of the shield is symmetric about an axis. What is the total area of the shield ?

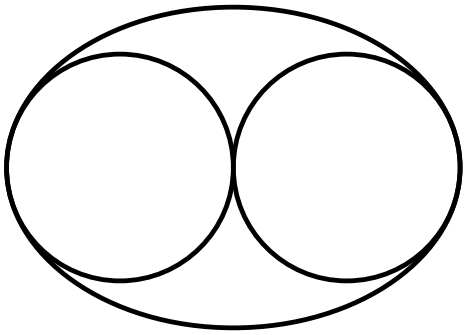


4. **Degree or gradian**

   Laurent did a trigonometry homework in which he needed to use his calculator to calculate the sine of a certain angle a, in degrees, lying between 0 and 360 degrees inclusively. Laurent did not realize that his calculator was not in degrees but in gradian (there are 100 gradians in a right angle). Nonetheless, he obtained the right answer. What was the value of the angle a, in degrees ? List all the possibilities.

5. **Die-duction**

   For $n \geq 1$, let $D_n$ be a die having $n$ faces with the numbers $0, 1, \ldots, n-1$. If the die is thrown, then there is an equal chance that each of these numbers will come up. The dice are thrown in the following manner. The die $D_n$ is thrown first. If 0 comes up, the process stops, otherwise $D_k$ is thrown next, where $k$ is the value obtained on the previous throw. This process is repeated until it reaches 0. If one calculates the sum of all the numbers that come up in this way, what is the expected value on average if one starts by throwing $D_6$ ?

6. **Enlipsed**

   Two circles of radius 1 are placed side by side. Then these two circles are encapsulated by the smallest ellipse possible of length 4. The ellipse only touches the circles at its two ends. What is the width of the ellipse ?