Concours de l'Association mathématique du Québec
Niveau collégial

Wednesday, February 20, 2019

TO THE COMPETITORS :

This is a competition rather than an examination; it is therefore natural that you may find certain questions difficult and that you may not be able to answer them all. Marking shall be done confidentially and will take into account various elements : approach, precision, clarity, rigour, originality and, in the case of a partial answer, a sketch of a viable solution.

We thank you and congratulate you for your interest in mathematics. Good luck!

Note : *The use of any kind of calculator is prohibited.*

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1. **Scary 13**

   Alexander has 100 cards numbered from 1 to 100. He wants to select a set of cards such that no pair of cards in his set has a sum divisible by 13. What is the least amount of card he will have to throw out for this to happen with the remaining cards?

2. **Perimeter**

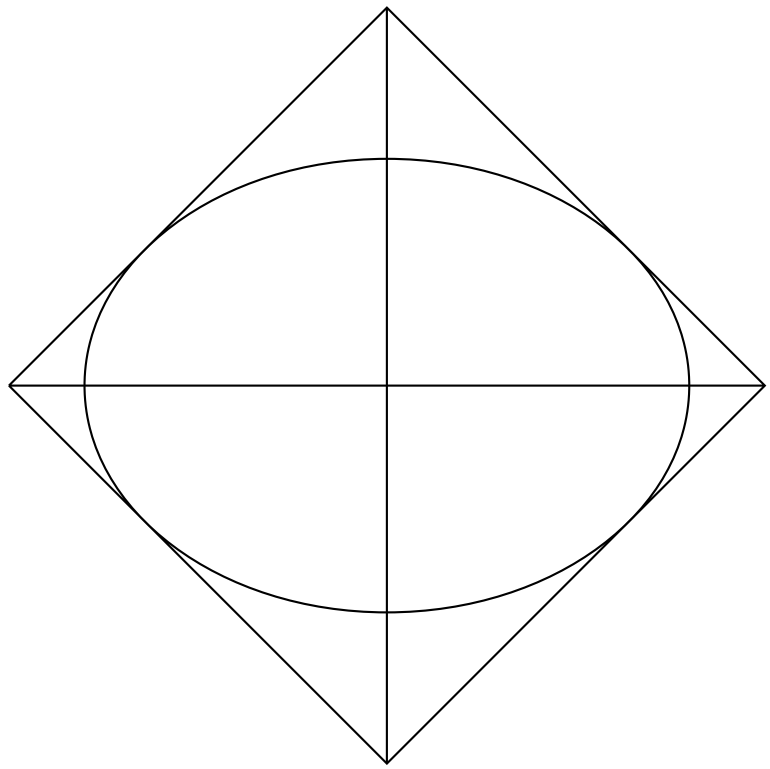
   Consider a right triangle with sides x, y, z and with area A. Given that

   $$\frac{x^4 + y^4 + z^4}{8} = 64 - A^2$$

   and that $x + y + z = 4A$,find the perimeter of the triangle.

3. Ellipse

Consider an ellipse with major axis 4 cm and minor axis 3 cm. Those axes are aligned on the diagonals of a square. The ellipse is also tangent to the square in four points. Find the area of the square.



4. A field of variables

Let x_0, x_1, x_2,...x_{2019} be positive or zero real numbers. Give the maximum value of x_0 such that

$$\sum_{n=0}^{2019} x_n = x_0 + x_1 + x_2 + x_3 + x_4 + \ldots = 1 \tag{1}$$

$$\sum_{n=0}^{2019} n x_n = \qquad x_1 + 2x_2 + 3x_3 + 4x_4 + \ldots = 1 \tag{2}$$

$$\sum_{n=0}^{2019} n^2 x_n = \qquad x_1 + 4x_2 + 9x_3 + 16x_4 + \ldots = 2. \tag{3}$$

5. A cube of cubes

All six faces of a cube are covered with a blue paint. The cube is broken into n^3 small cubes and those are placed in an opaque bag. A small cube is then randomly selected and thrown like a die.

What is the probability that the top face shows blue ?

6. Hyperroot

A real number t is called an hyperroot of c if $t > 0$ and $t^t = c$. For instance, 2 is the hyperroot of 4 because $2^2 = 4$ and 2 is the hyperroot of 27 because $3^3 = 27$.

Show that the hyperroot of 2 is an irrational number.